The first result of the morphological coupling of the breaking wave model

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Motivation

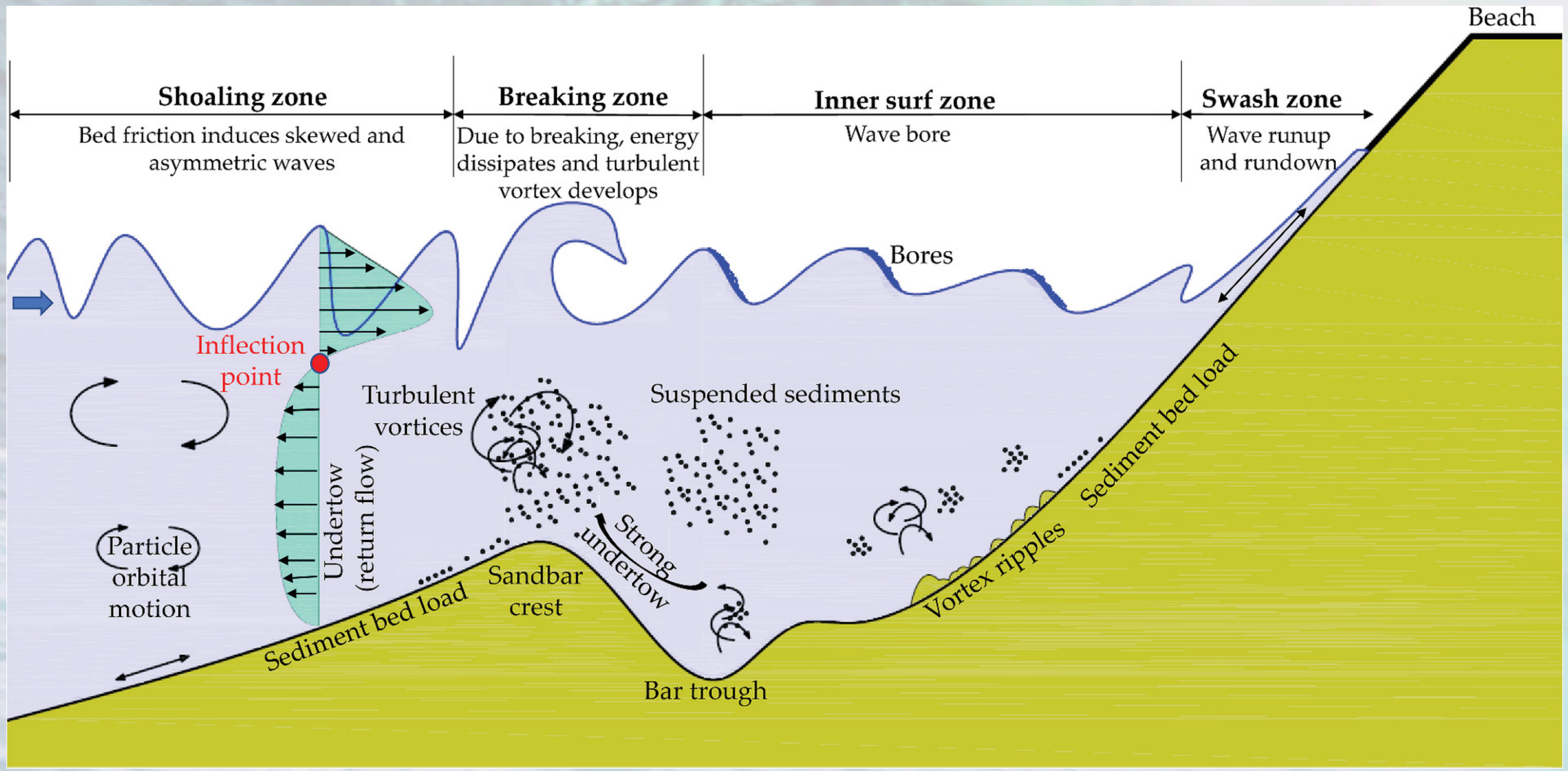
Coastal erosion is a worldwide issue → Understand sediment transport mechanism and the wave dynamic

Beach at Biscarrosse, Southwestern France

The beach on Tainan's Golden Coast, Taiwan, disappeared

Adapted from: [Le monde: By 2100, coastal erosion will have affected thousands of buildings in France]
[Taiwan: Saving the disappearing coastline]

Near-shore wave dynamic



Elsayed, S. M., et al. (2022) “Nonhydrostatic numerical modeling of fixed and mobile barred beaches: Limitations of depth-averaged wave resolving models around sandbars."

- In shoaling zone, asymmetric wave
- In breaking zone, turbulence is generated and large amounts of energy are released

↓

Entrain sediment into the fluid

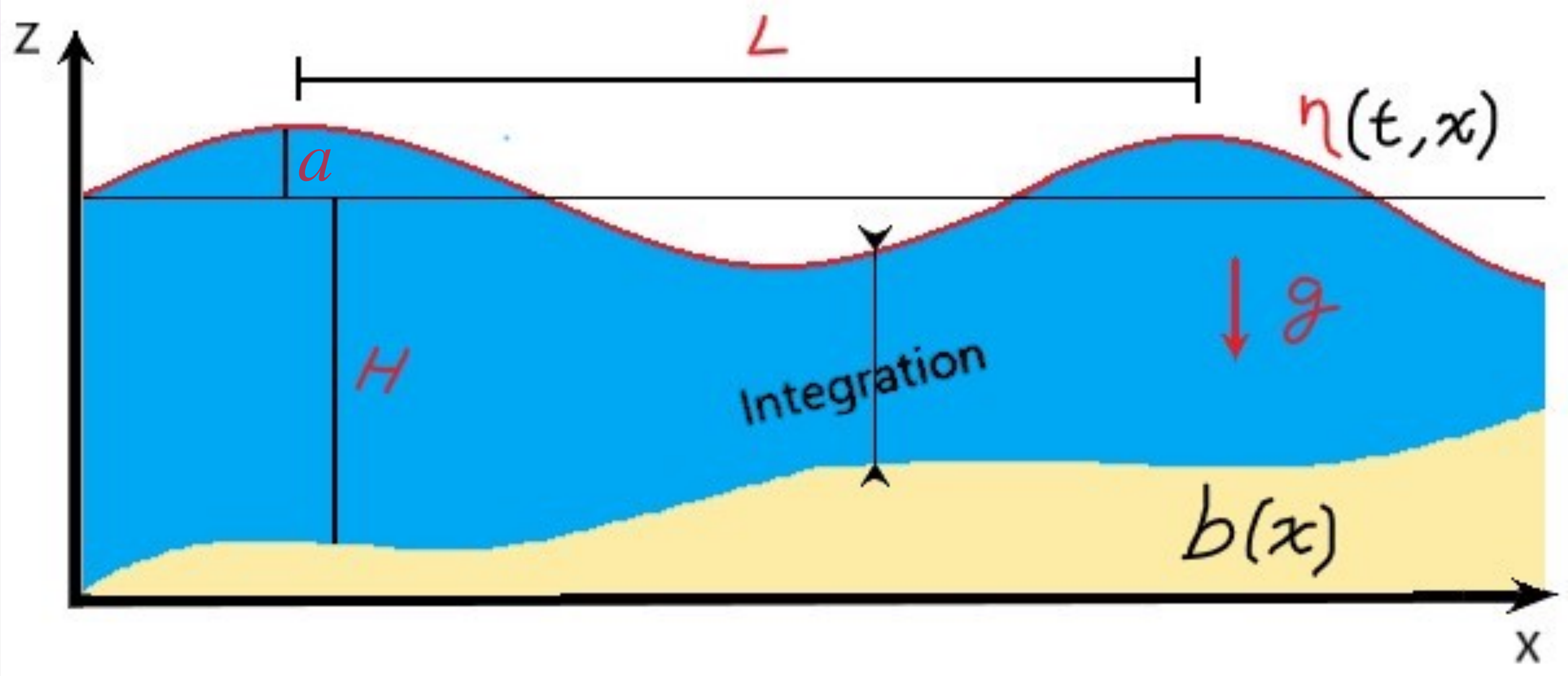
- Undertow transports sediment in the off-shore direction

modeling breaking wave becomes a priority before studying sediment transport

Outline

1. Breaking wave model
   - Model derivation and analysis
   - Numerical validation
2. Morphodynamic coupling
   - Numerical scheme
   - LIP experiment

Depth-averaged model



Navier-Stokes equation

- ✔ Full resolution of the interior of fluid
- ✘ Computational costly

- Integrating over the depth
- Asymptotic analysis

$$\frac{\partial h}{\partial t} + \mathrm{div}(h\boldsymbol{U}) = 0$$

Mass eq.

$$\frac{\partial h\boldsymbol{U}}{\partial t} + \mathrm{div}\left(h\boldsymbol{U}\otimes\boldsymbol{U} + \frac{gh^2}{2}\mathbf{I} + p_N\right) = 0$$

Momentum eq.

$\delta = (H/L)^2$ (Shallowness/dispersion)

$\varepsilon = a/H$ (Nonlinearity)

$$\dot{h} = \frac{Dh}{Dt} = \frac{\partial h}{\partial t} + U\frac{\partial h}{\partial x},\ \ddot{h} = \frac{D^2h}{Dt^2}$$

model	**NSWE** $O(\delta)$	**SGN** $O\left(\delta^2\right)$
Pressure	$p_N = 0$	$p_N = h^2\ddot{h}/3$
Type	Hyperbolic	Dispersive

Breaking wave : Energy dissipation

Classic models do not take breaking into account.

Their derivation within a conservative framework ⟶ no natural energy dissipation mechanisms due to wave breaking

Classic methods are:

- Extra terms in the mass and/or the momentum equation to provide a necessary dissipation.
- Switching or hybrid methods (natural dissipation through the shock)

Recent approach is to use the enstrophy (Teshukov, 2007)
Richard & Gavrilyuk, **2015**;
Gavrilyuk, Chesnokov & Liapidevskii, **2016**

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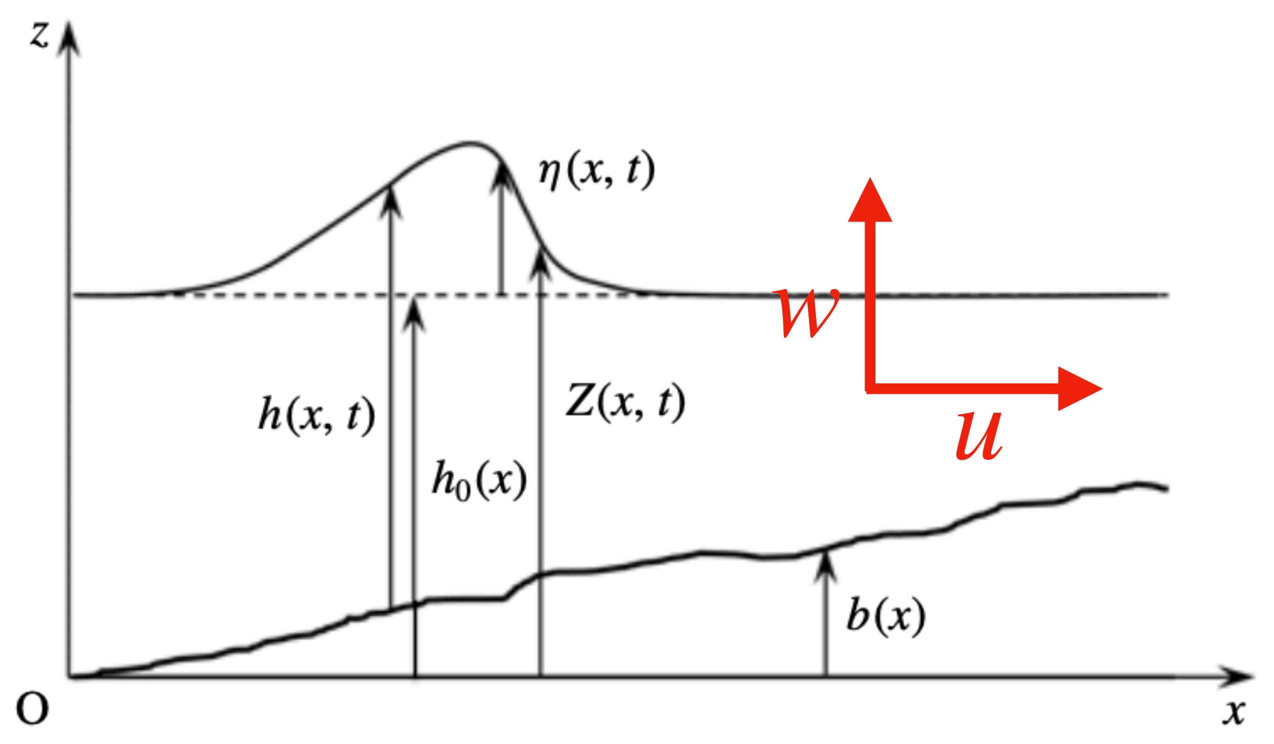
Model assumptions

About breaking

u
- $\bar{u}$
  - U — Mean velocity (uniform over the depth)
  - u' — Able to describe turbulence
- Residual term — Contribute to energy dissipation due to wave breaking (with turbulent viscosity hypothesis)

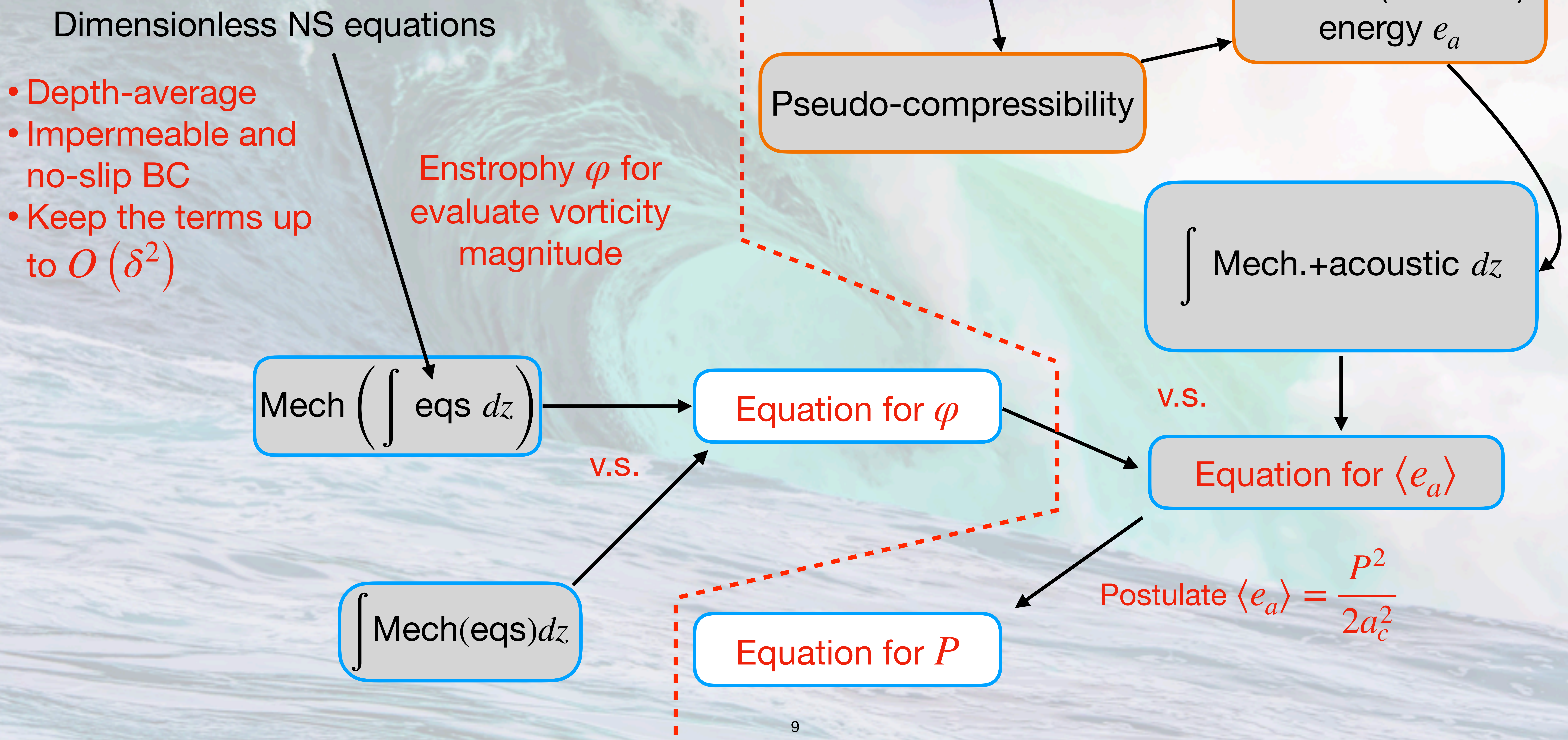
The assumption of both NSW and SGN model

Same assumption of the breaking wave model developed by Kazakova & Richard, **2019**



Breaking waves model

Model derivation



Breaking waves model

Final dimensional equations under mild slope condition

$$\tilde{\varphi} := \frac{\langle \tilde{u}'^2 \rangle}{\tilde{h}^2} \equiv \frac{1}{\tilde{h}^3} \int_{\tilde{b}}^{\tilde{\eta}} \tilde{u}'^2 d\tilde{z}$$

$$\frac{\partial h}{\partial t} + \frac{\partial hU}{\partial x} = 0$$

$$\frac{\partial hU}{\partial t} + \frac{\partial}{\partial x}\left(hU^2 + \frac{gh^2}{2} + h^3\varphi + hP \right) = \frac{\partial}{\partial x}\left(2\nu_T h \frac{\partial U}{\partial x} \right) - gh\frac{\partial b}{\partial x}$$

$$\frac{\partial hW}{\partial t} + \frac{\partial hUW}{\partial x} = \frac{3}{2}P + 3\nu_T \frac{\partial U}{\partial x}$$

$$\frac{\partial hP}{\partial t} + \frac{\partial hUP}{\partial x} = -a_c^2 \left(h\frac{\partial U}{\partial x} + 2W \right)$$

$$\frac{\partial h\varphi}{\partial t} + \frac{\partial hU\varphi}{\partial x} = -\frac{2}{h}\langle P^r \rangle + \frac{4\nu_T}{h}\left(\frac{\partial U}{\partial x} \right)^2 - \frac{8\nu_T W}{h^2}\frac{\partial U}{\partial x}$$

- The conservative part of the model is hyperbolic if $h > 0$
- Asymptotic dispersive properties to the SGN equation when $a_c \to \infty$

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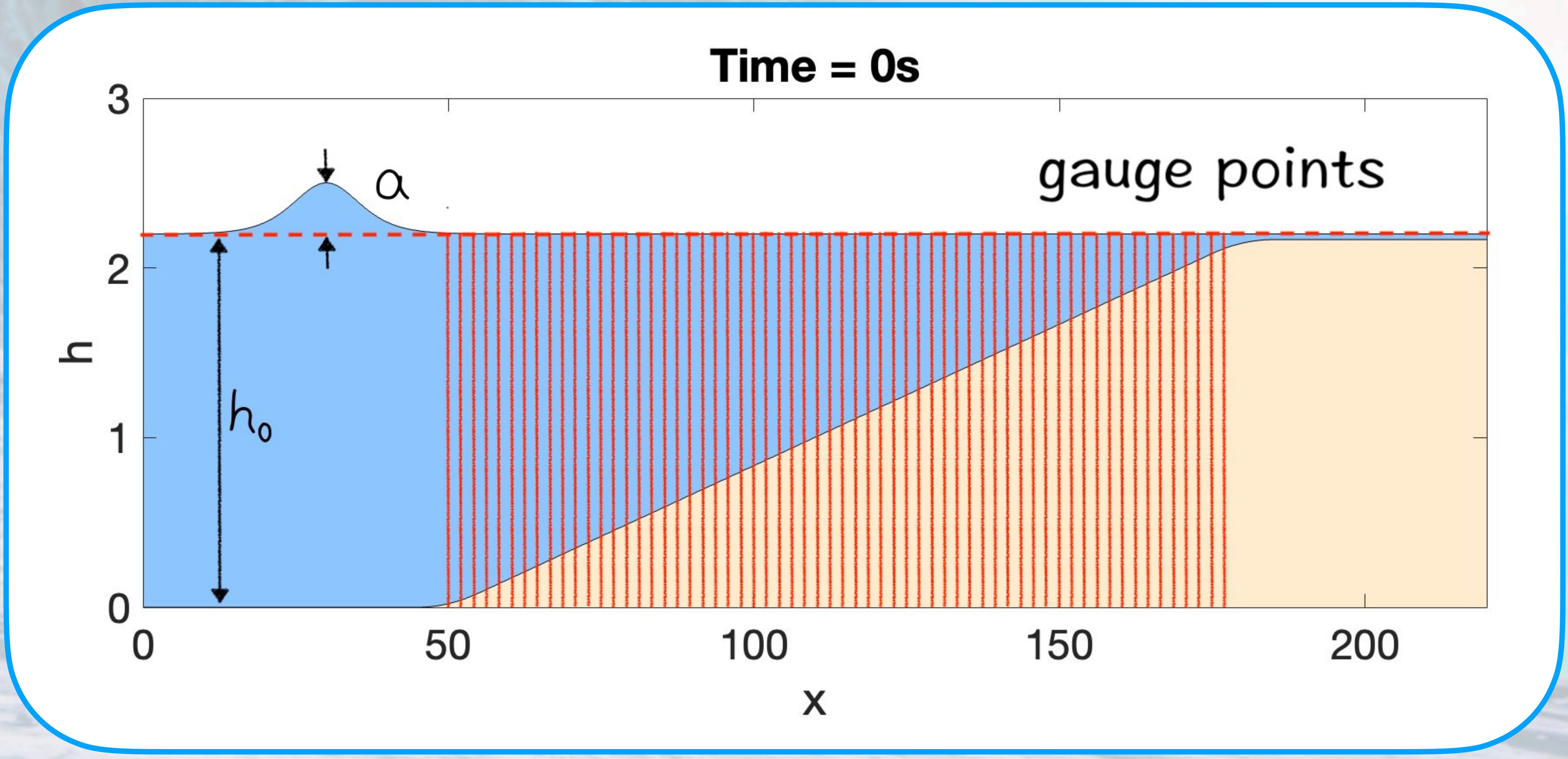
Numerical validation

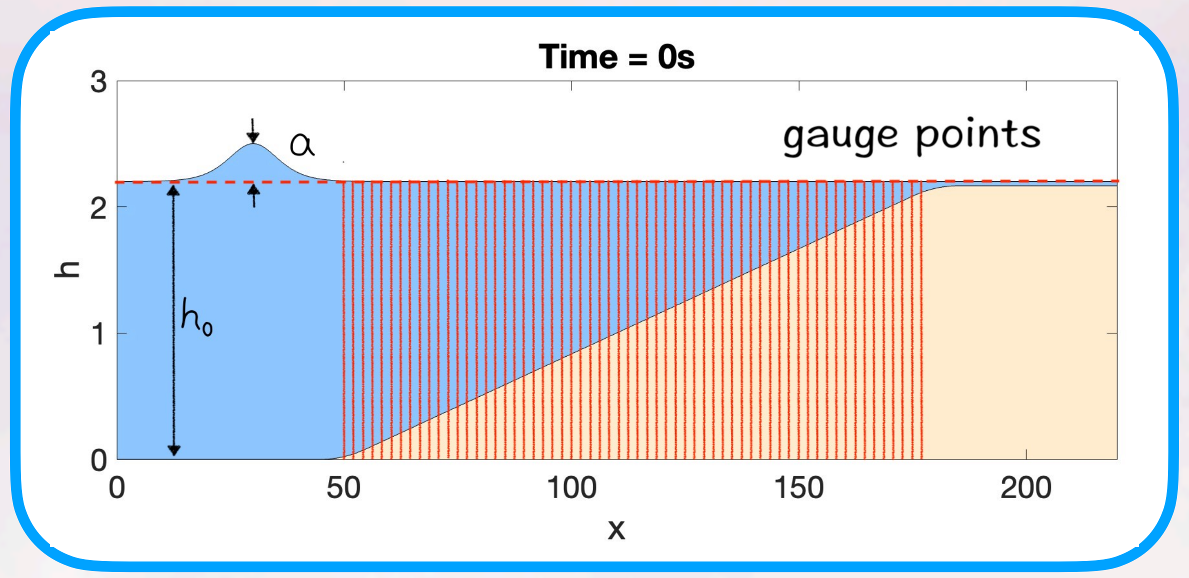
Experiment of Hsiao, 2008

Solitary waves with different water depths h_0 and non-linearities $\varepsilon = a/h_0$ propagate onto a slope with $\tan \beta = 1/60$



Numerical validation

Experiment of Hsiao, 2008; Gauge point comparison of Trial 25 & 41



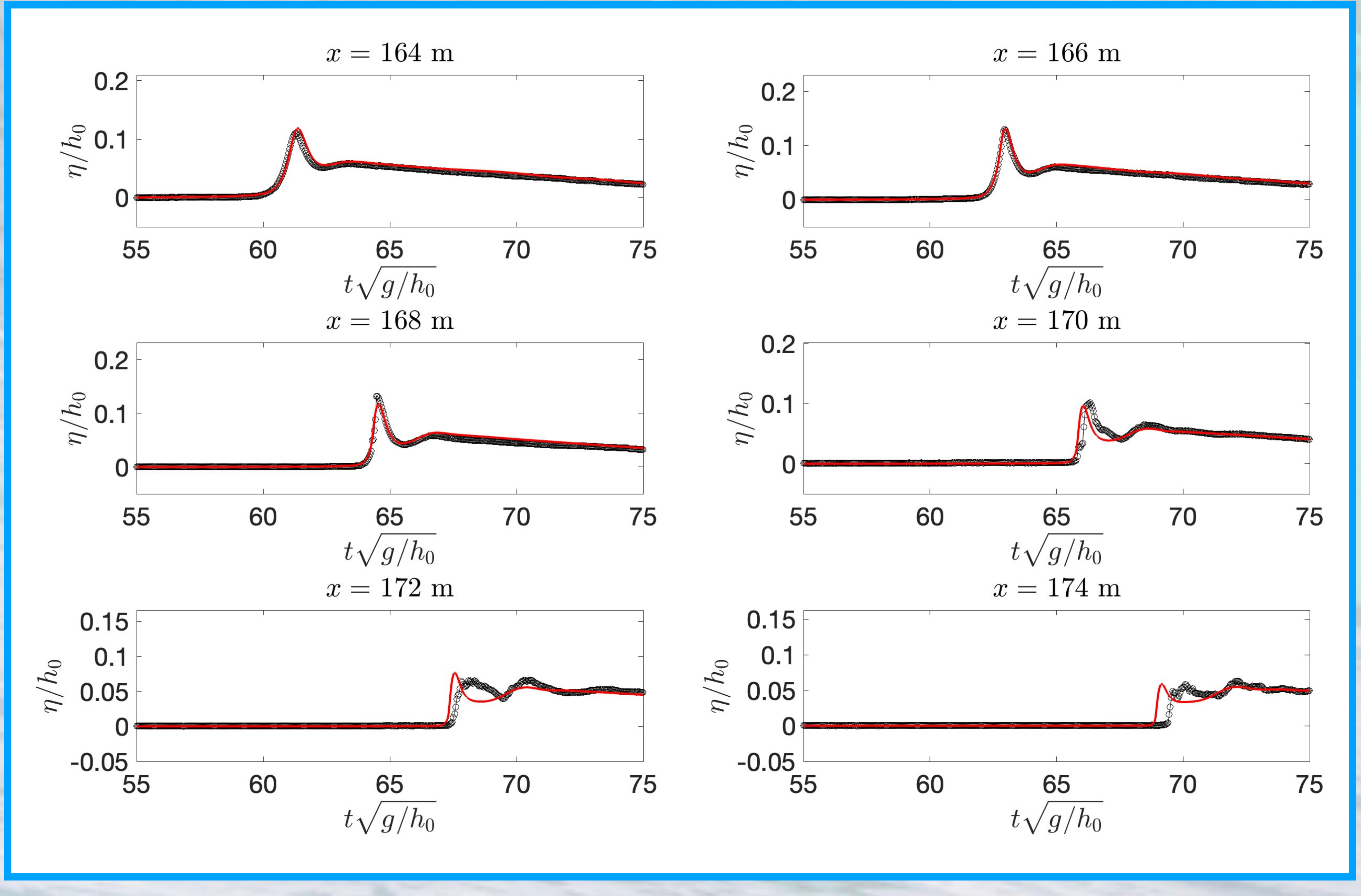
Trial 25
$h_0 = 2.2$ m
$\varepsilon = 0.054$

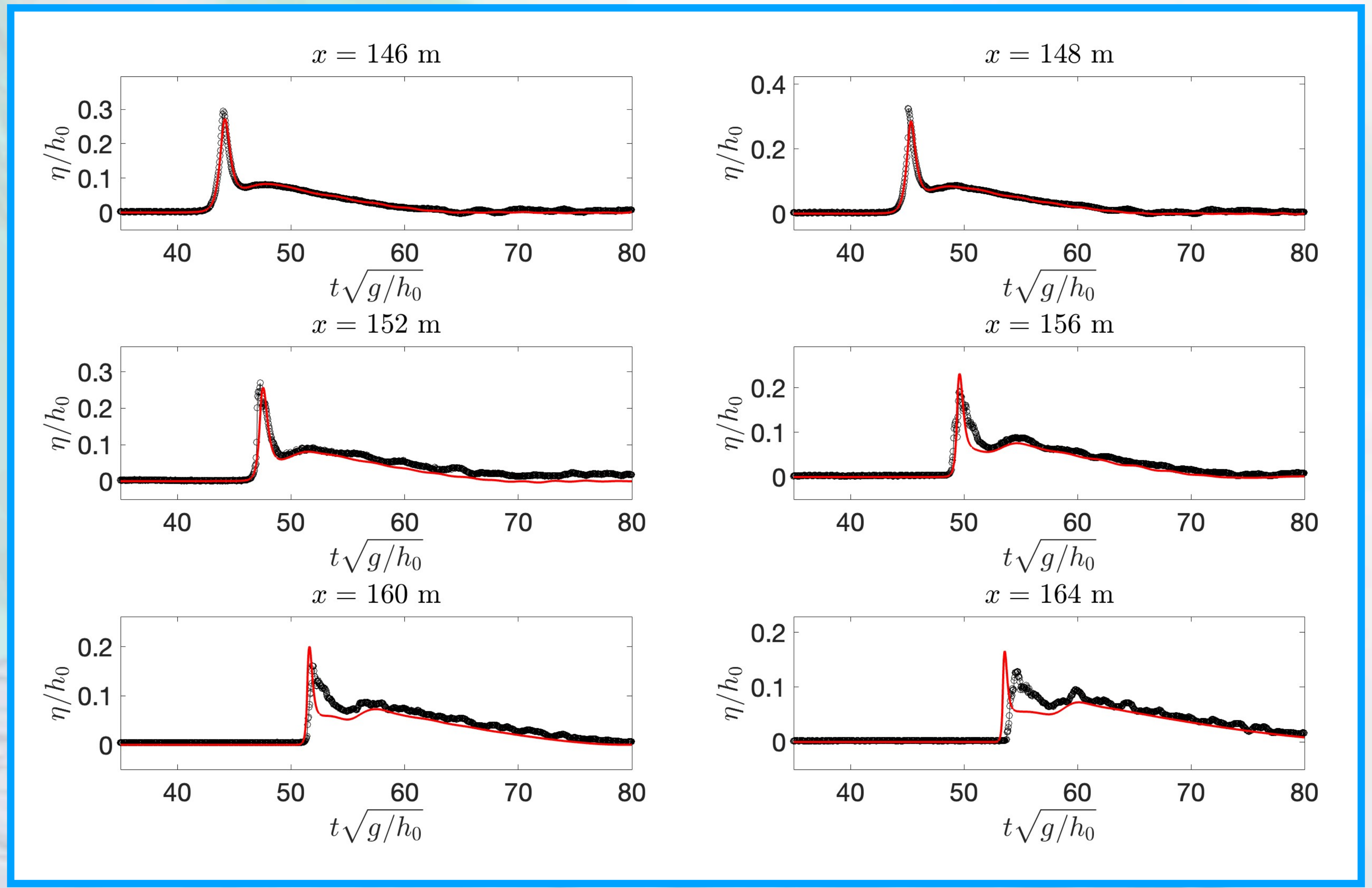
Breaking position:
Real:167 m
Numerical:166.7 m

Trial 41
$h_0 = 2.2$ m
$\varepsilon = 0.137$

Breaking position:
Real:148 m
Numerical:148.5 m

Amplitude



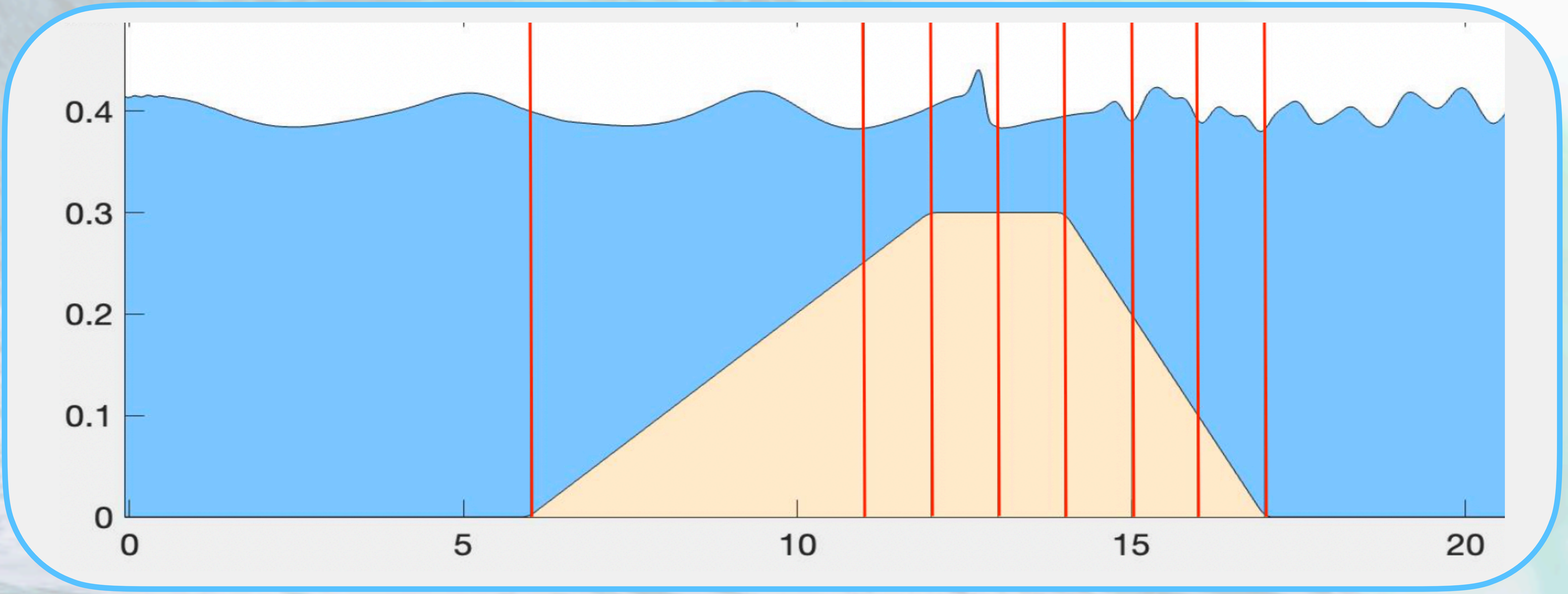


Time Series

Numerical validation

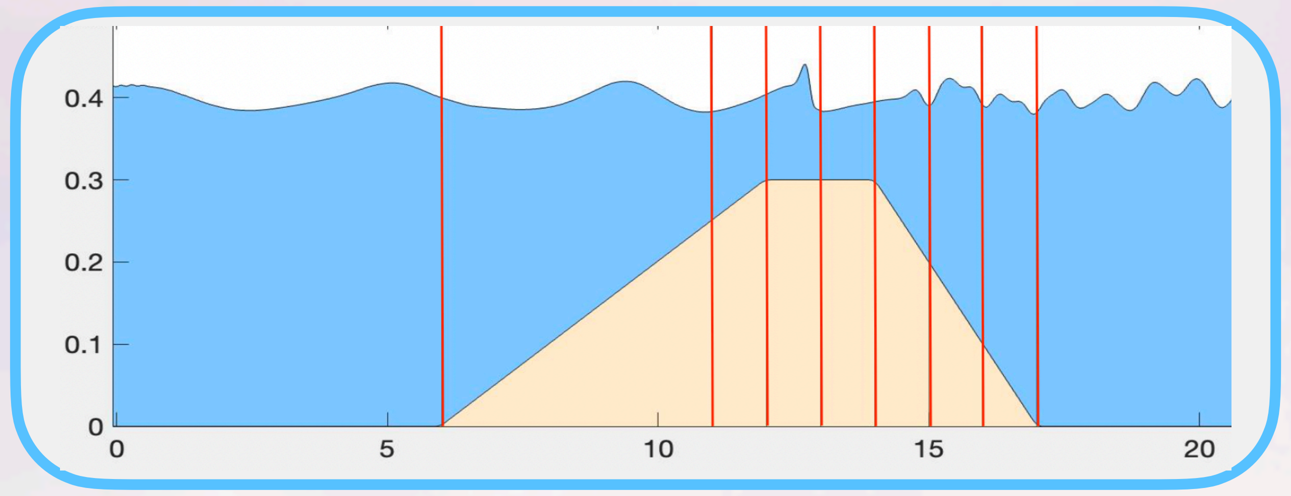
Experiment of Beji & Battjes, 1993

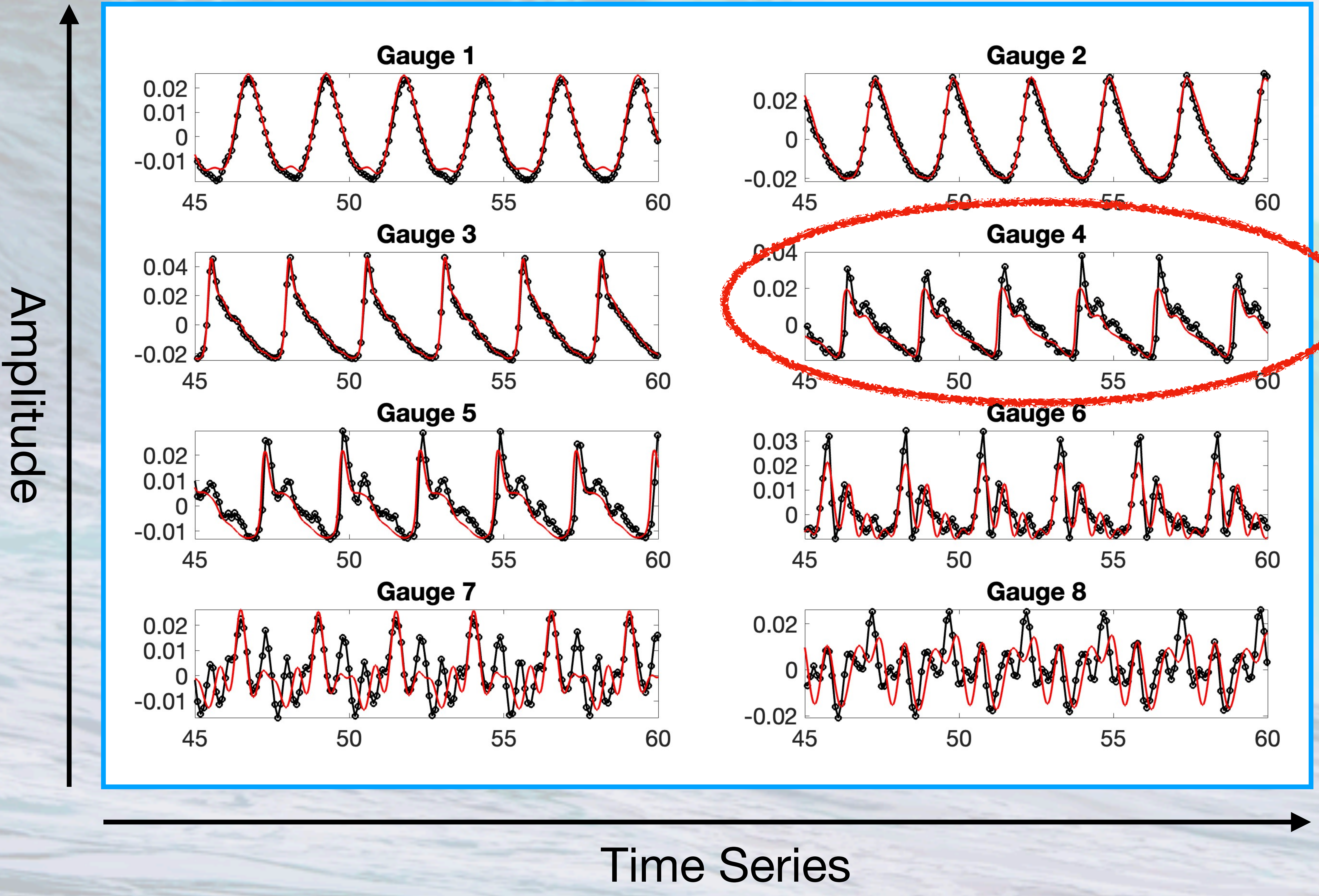
Regular and irregular wave trains with water depth $h_0 = 0.4$ m enter the domain from the left and propagate onto a submerged bar.



Numerical validation

Experiment of Beji & Battjes, 1993; Gauge point comparison





True breaking position lies between gauge 3 and 4

Dispersive property can be improved for the regime with strong dispersive effect (**Bonneton, 2011**)

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Motivation

European Large Installation Plan (LIP) experiments was carried out in Delft Hydraulics's Delta Flume.

LIP experiment condition: 13h~18h

- **Marchesiello et. al., 2022; Shafiei et. al., 2023** use CROCO to run the LIP experiments
  → Does examine a long term simulation but being a 3D wave-resolving model

- **Mancini et. al., 2020** couple XBeach with sediment transport equations
  → Depth-averaged model but doesn't examine a long term simulation

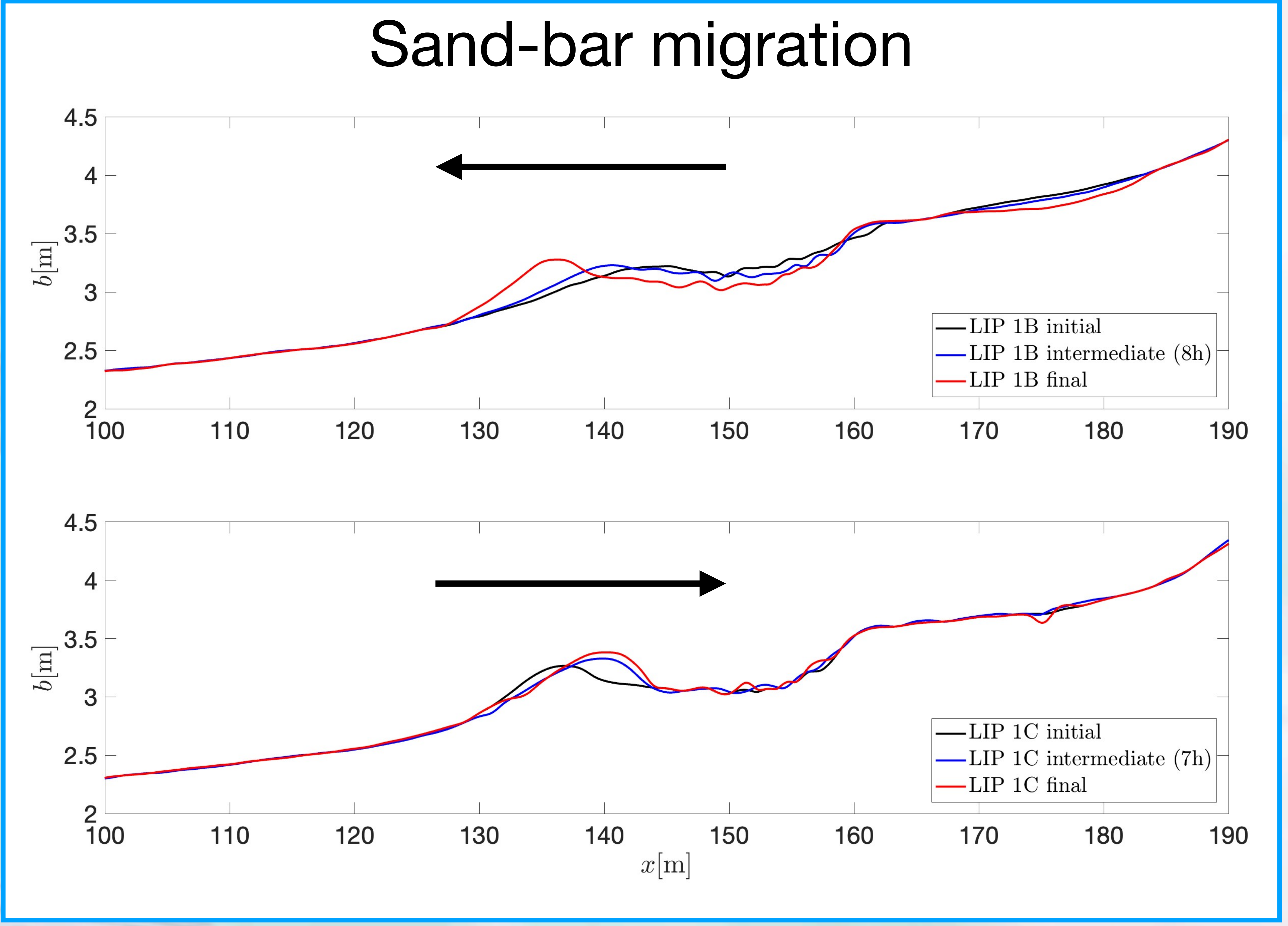
LIP Experiments

LIP-1B (More energetic)

- More energetic wave trains
- Effect of the suspended load dominant
- Create undertow
- Move sand-bar away from the shore

LIP-1C (Less energetic)

- Less energetic wave trains
- Effect of the bed load dominant
- Move sand-bar toward the shore



Goal: Does depth-averaged model capable to describe the undertow effect?

Morphodynamic equations

Equation for suspended-load (Concentration)

$$\frac{\partial hC}{\partial t} + \frac{\partial hUC}{\partial x} = E - D$$

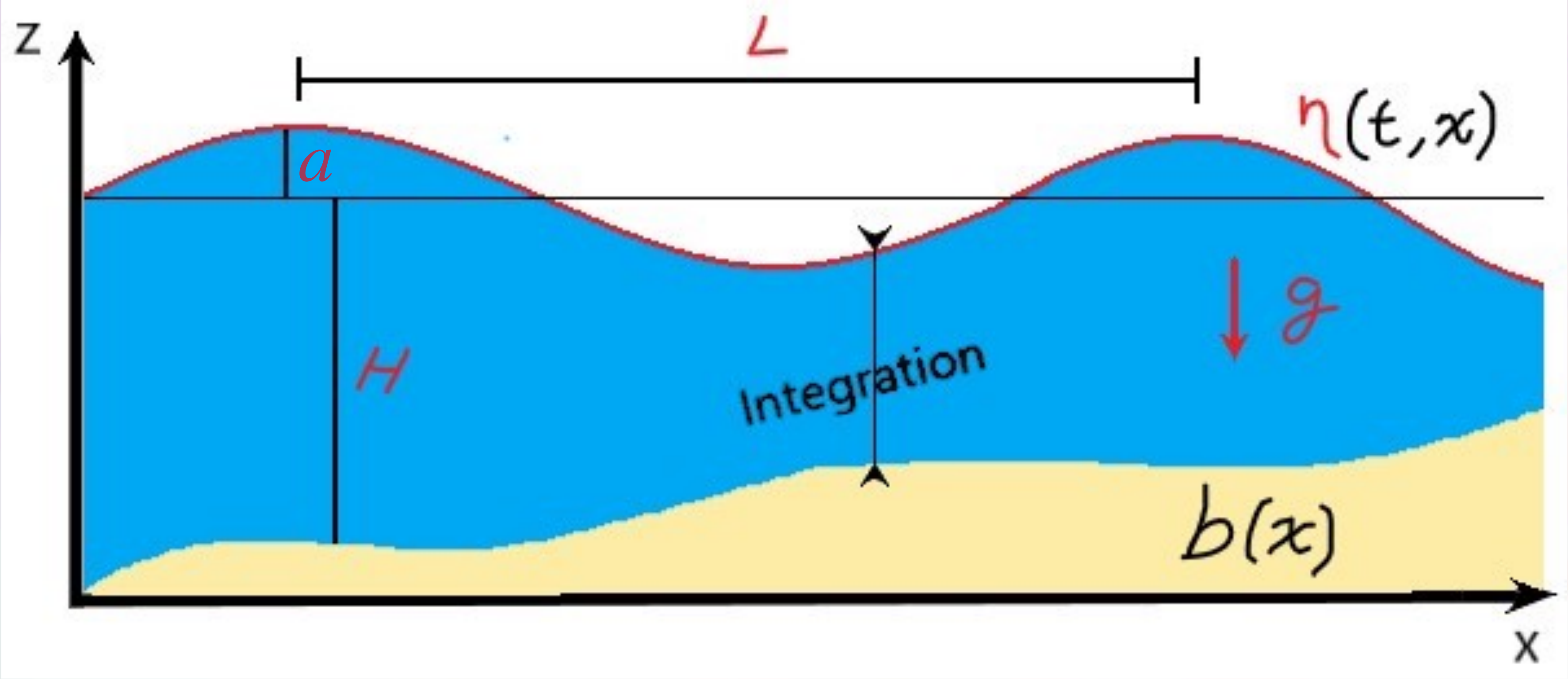
Equation for bed evolution (Exner-type)

$$\frac{\partial b}{\partial t} + \frac{\partial q_b}{\partial x} = D - E$$

- Simple
- Applicable in large scale sediment transport

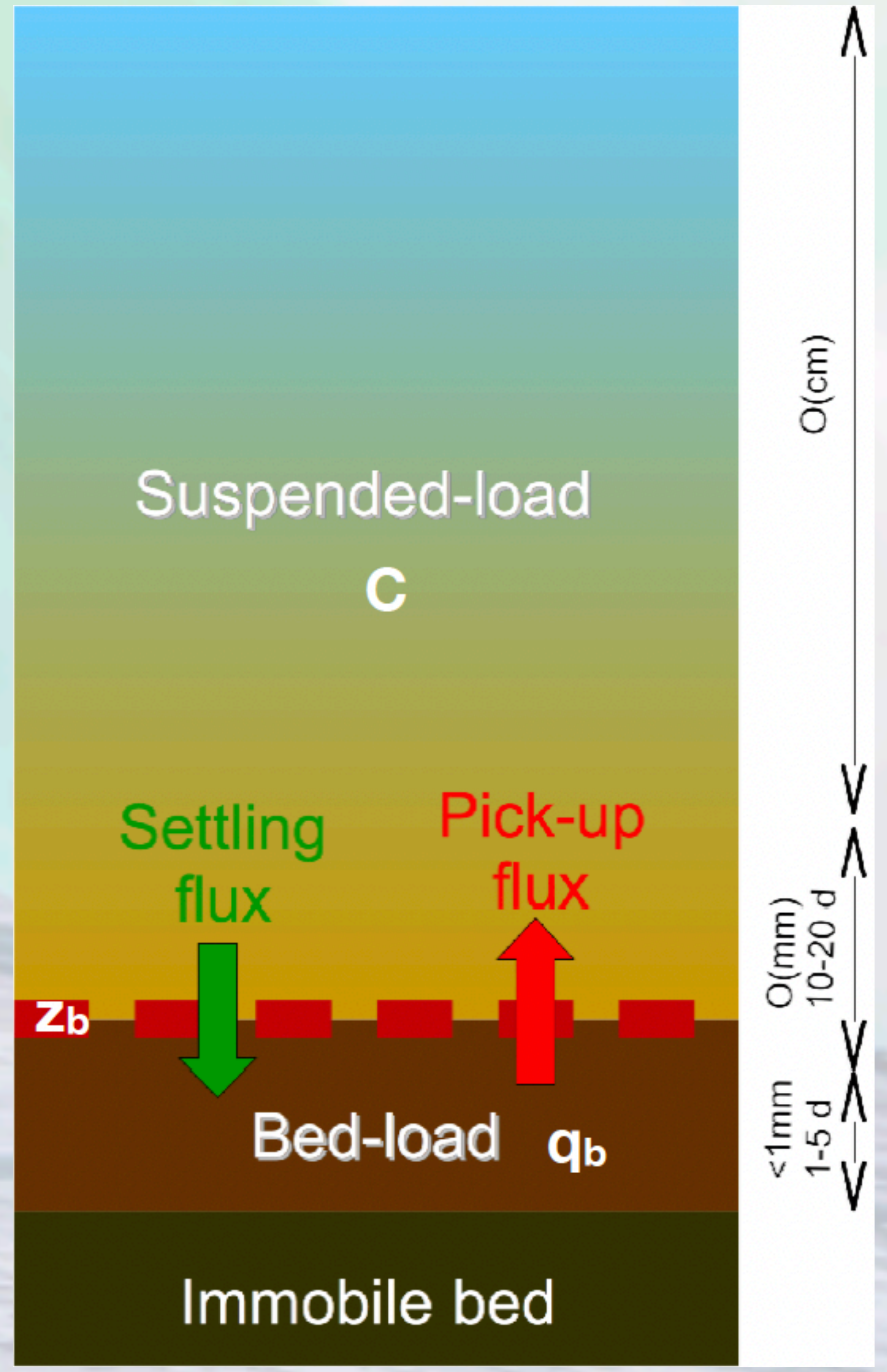
But

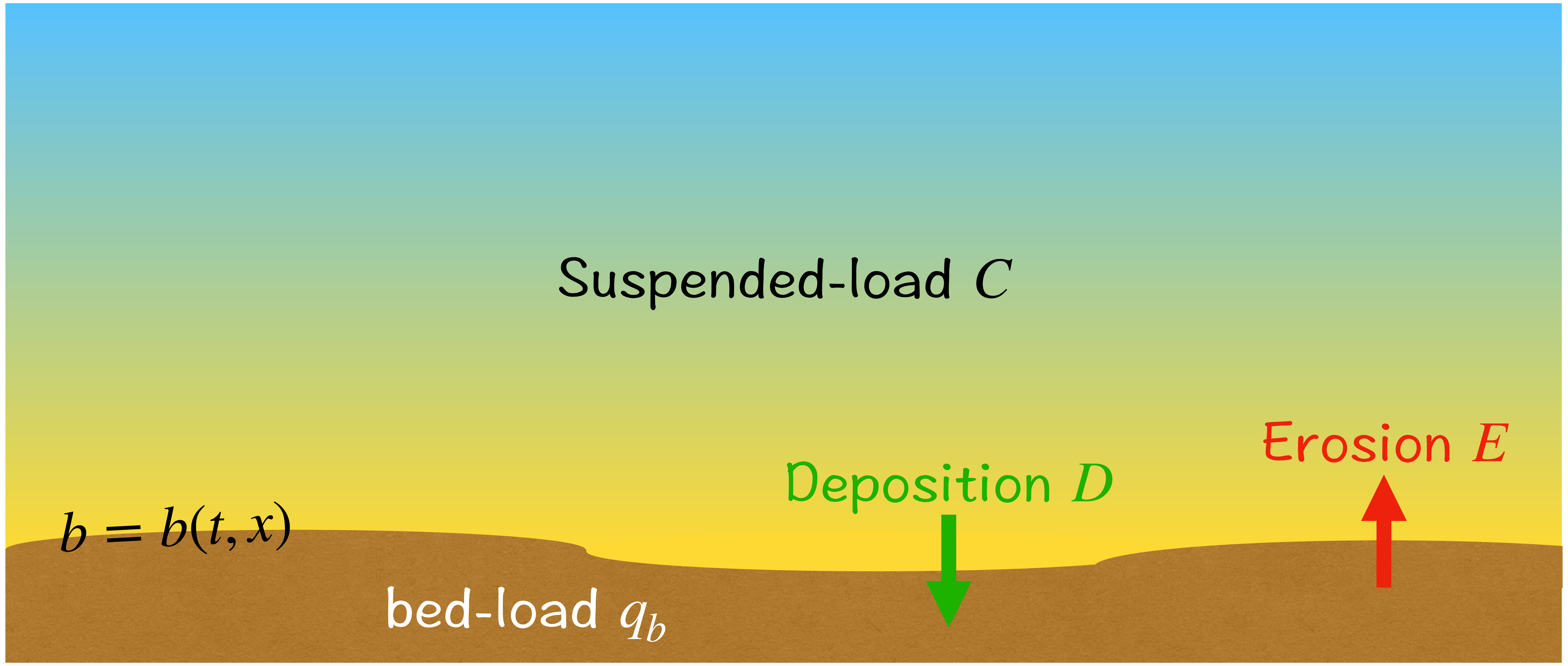
- Highly Empirical



E: sediment erosion rate

D: sediment deposition rate




Suspended-load C
Erosion E
Deposition D
$b = b(t, x)$
bed-load q_b

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Numerical scheme & challenges

Same scheme for hydrodynamic equations: IMEX ARS2(2,2,2)

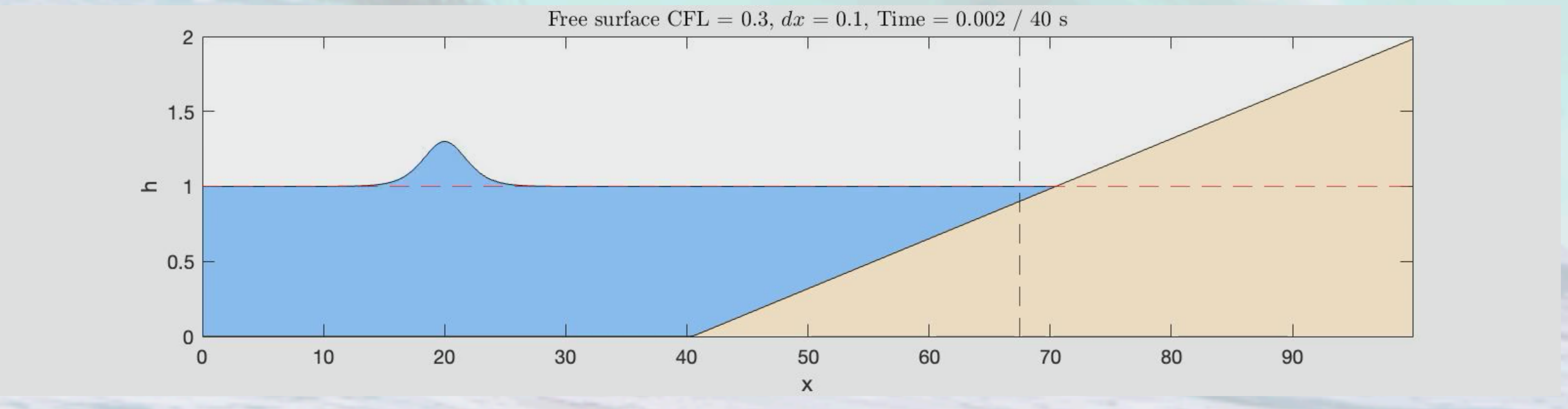
Hydrodynamic equation + equation of C

The non-oscillatory central (NOC) differencing schemes (V. Marieu, **2007**; T. Nagel, **2014**)

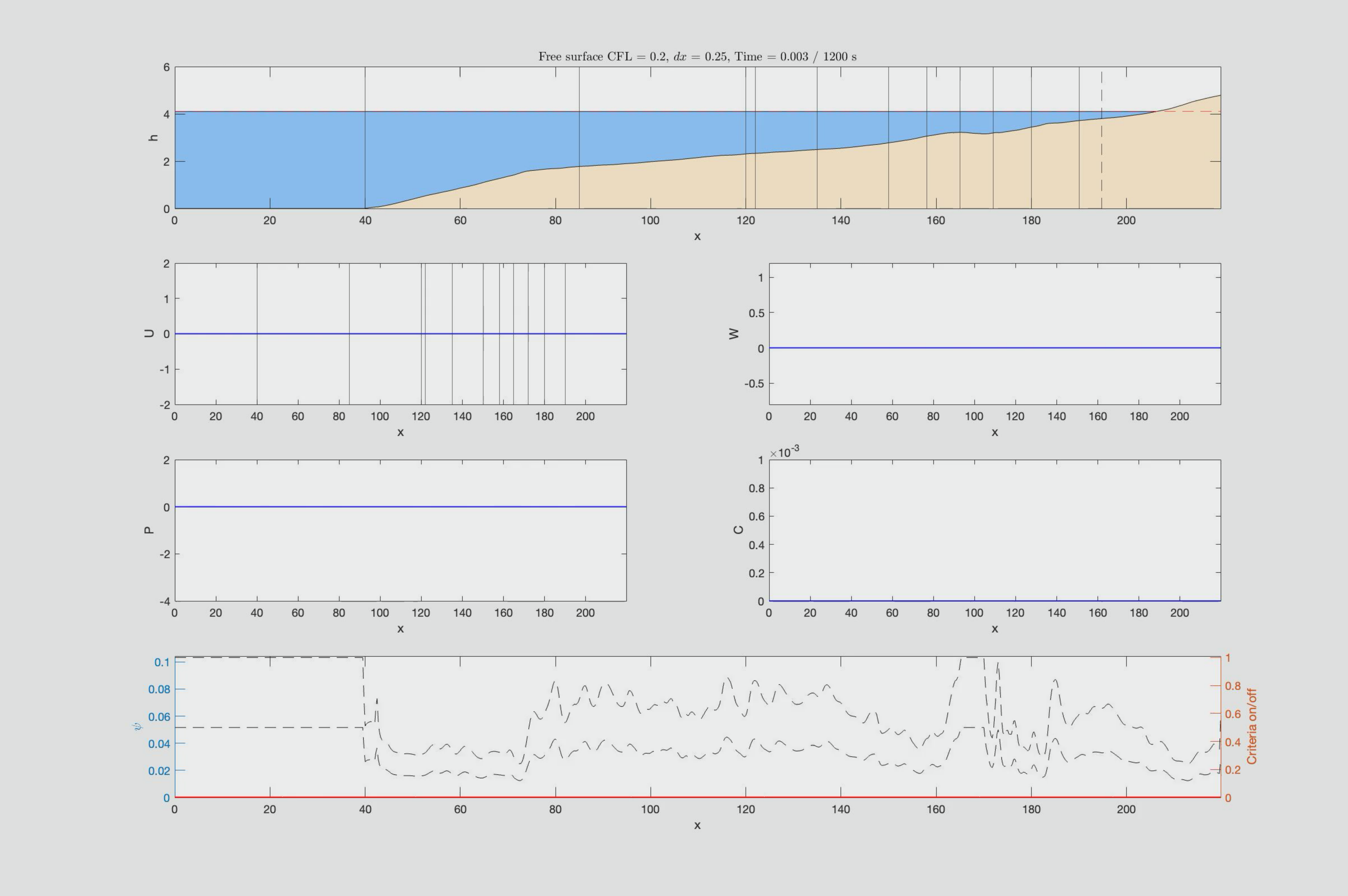
Bed evolution equation

Numerical challenges:

1. The generation of irregular wave trains
2. Wet-dry interface



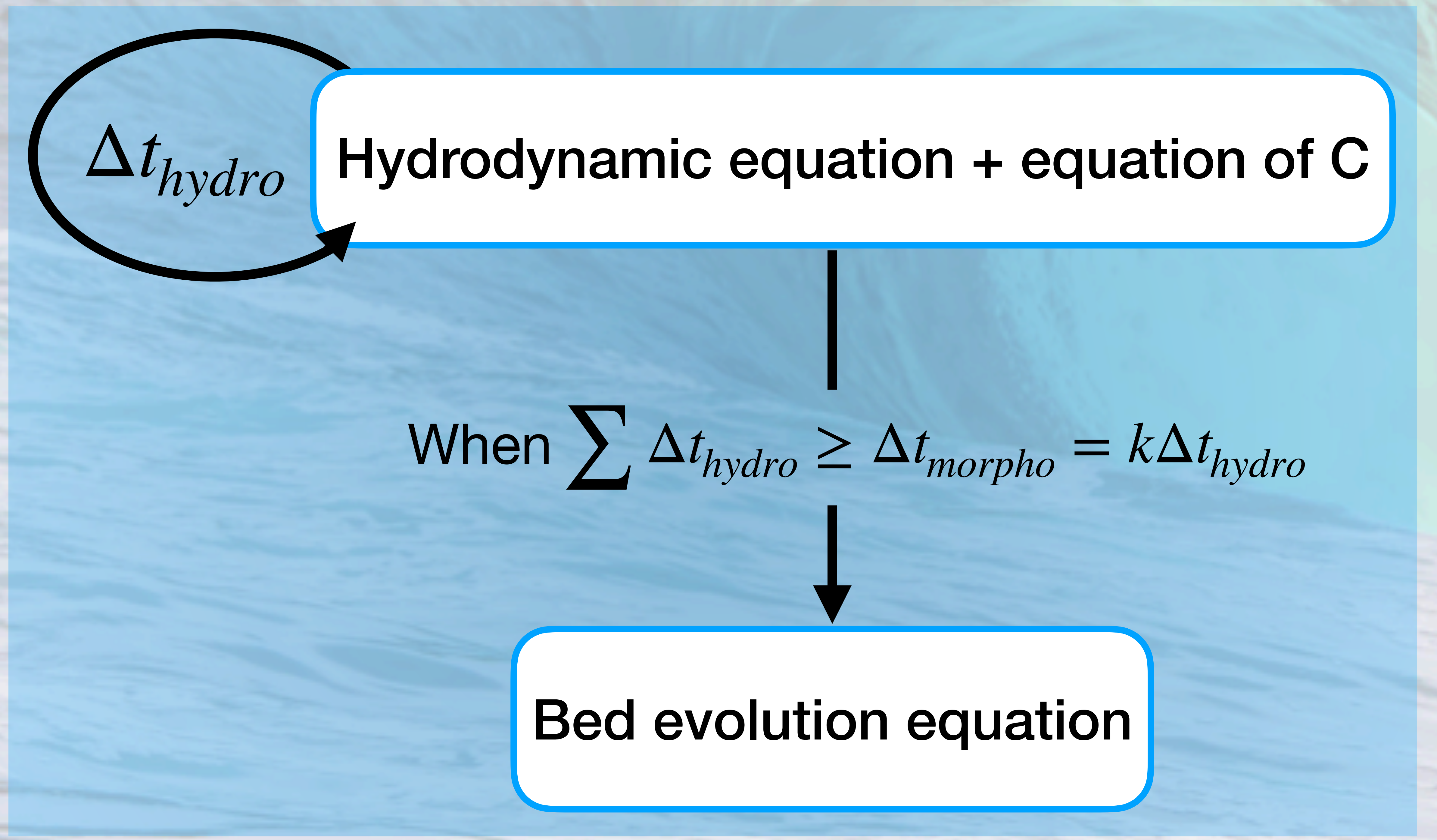
Numerical scheme & challenges



Different time scale

Morphodynamic evolution is much slower than the hydrodynamic processes → Assign two different time steps

Only update the bed evolution equation after multiple hydrodynamic time steps



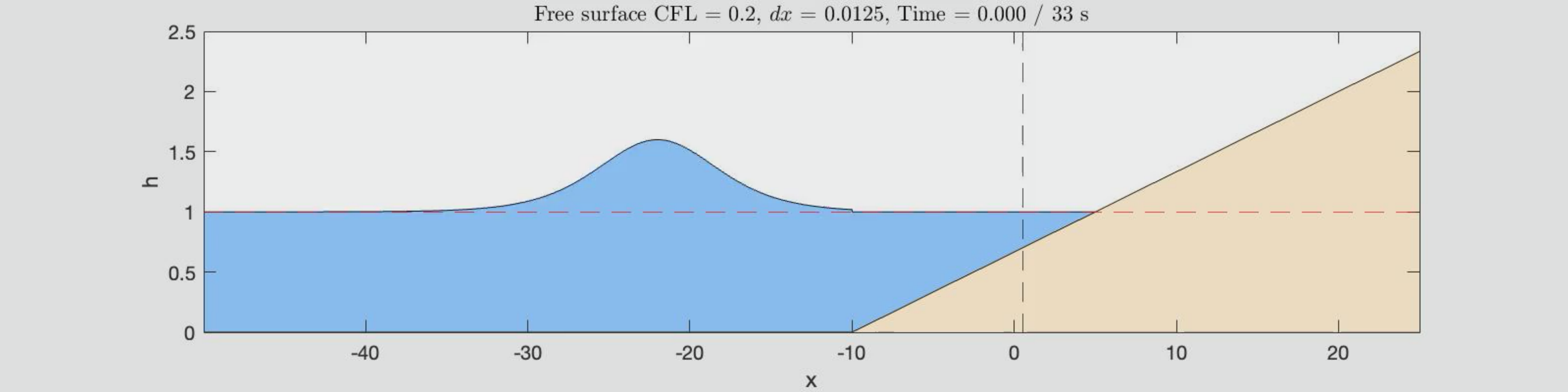
In the case of wave train: **Latteux, 1995**

$$\frac{\partial b}{\partial t} + f_m \frac{\partial q_b}{\partial x} = f_m (D - E)$$

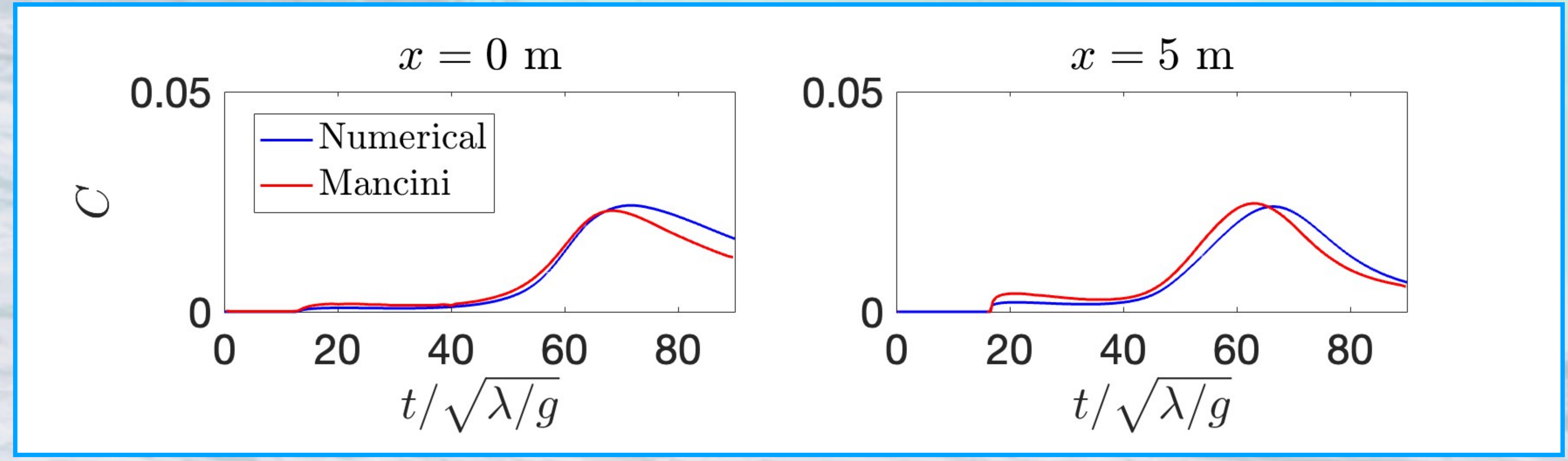
- Simulate f_m successive tides with 1 tide

Solitary wave run-up (Zhu & Dodd, 2015)

Model: NSW equation + equation of C + Exner equation

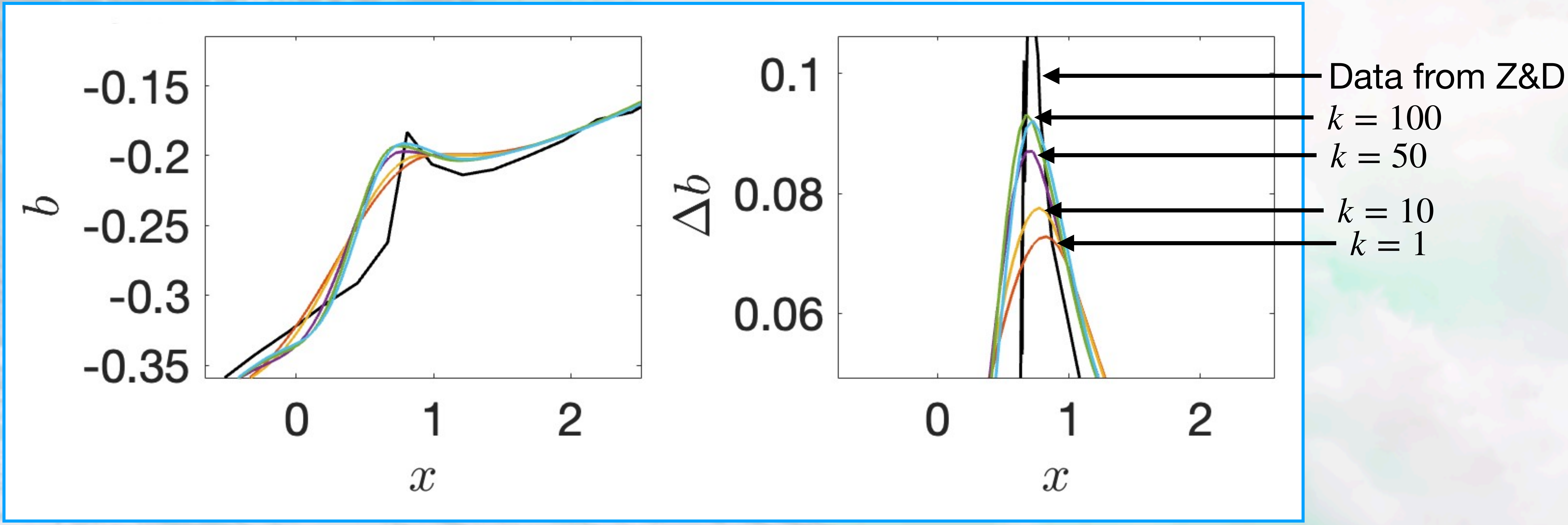


Time series comparison of C at $x = 0$ m and $x = 5$ m with Mancini, 2020



Bed profile comparison

Final bed profile with different choice of k



- Update bed less frequently gives a less diffusive result
- Choice of k should ensure the stability condition of the Exner equation

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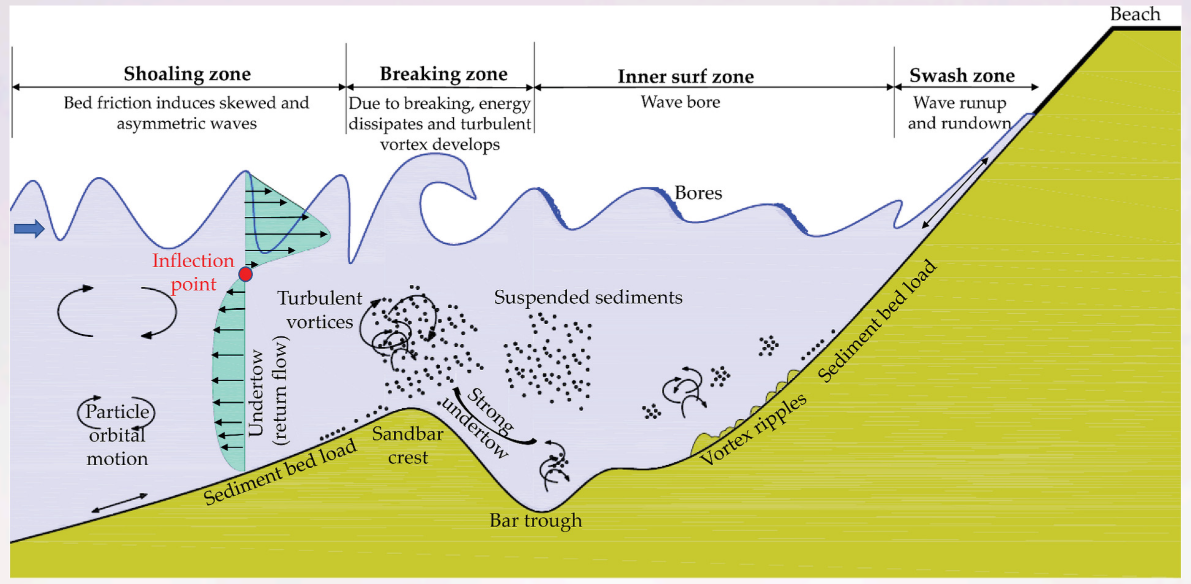
Link with wave breaking



Elsayed, S. M., et al. (2022)

The sediment erosion rate

$$E = \begin{cases} E_0(1-n_p)\left(\dfrac{|\tau_b| - \tau_{b,cr}}{\tau_{b,cr}}\right), & \text{if } |\tau_b| > \tau_{b,cr} \\ 0, & \text{otherwise} \end{cases}$$

The friction law takes into account the effect of wave-breaking

For TKE model:

$$\tau_b = C|u_b|\sqrt{u_b^2 + \phi^2(x_b)\alpha_{br}^2 k_b}$$

$\phi(x)$: Gaussian function centered around breaking point

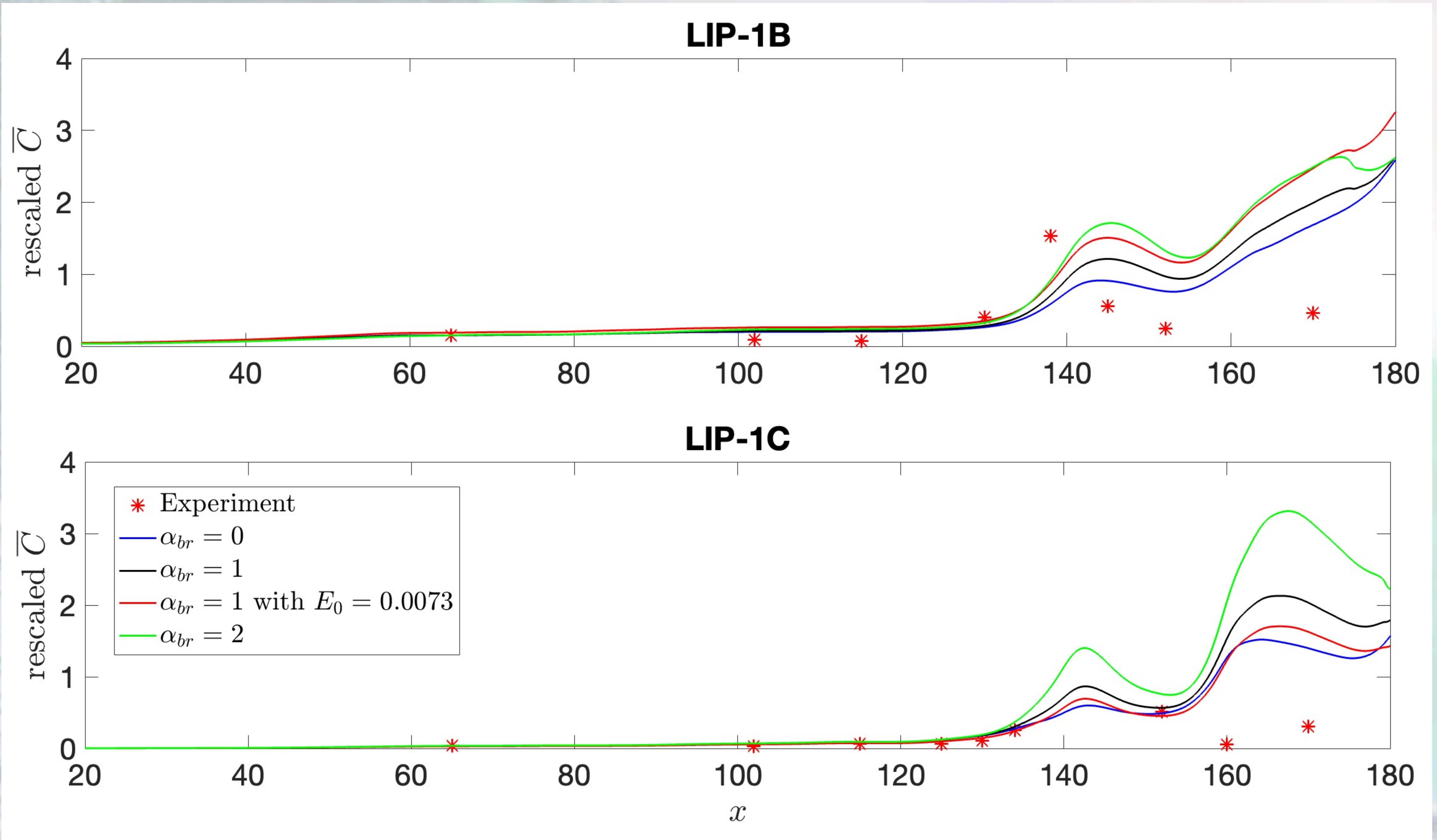
k_b : Turbulent kinetic energy

In my model:

$$\tau_b = C|U|\sqrt{U^2 + \alpha_{br}^2 \varphi}$$

φ : enstrophy

The effect of breaking on suspended load



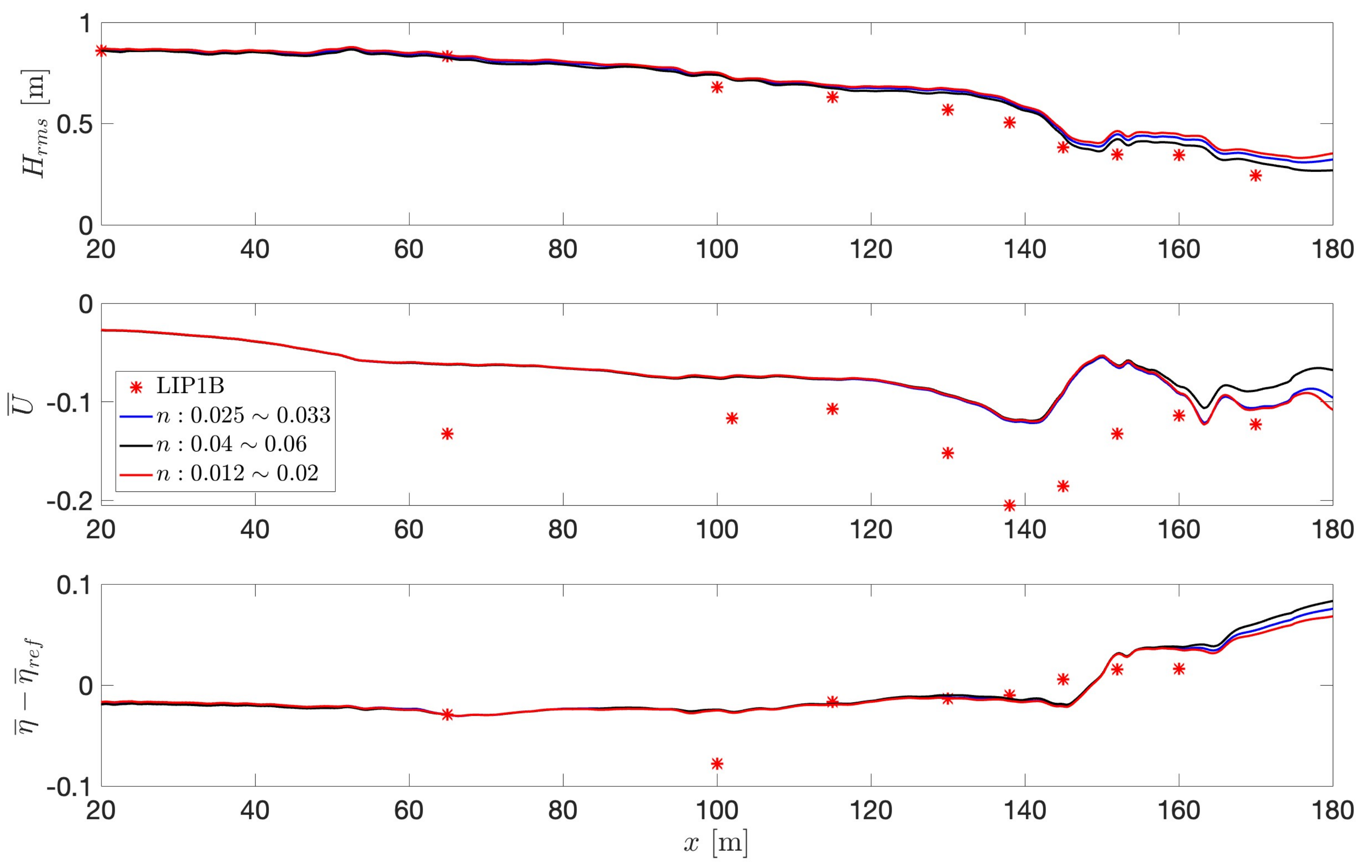
Undertow effect

H_{rms} : root-mean-square of the wave height

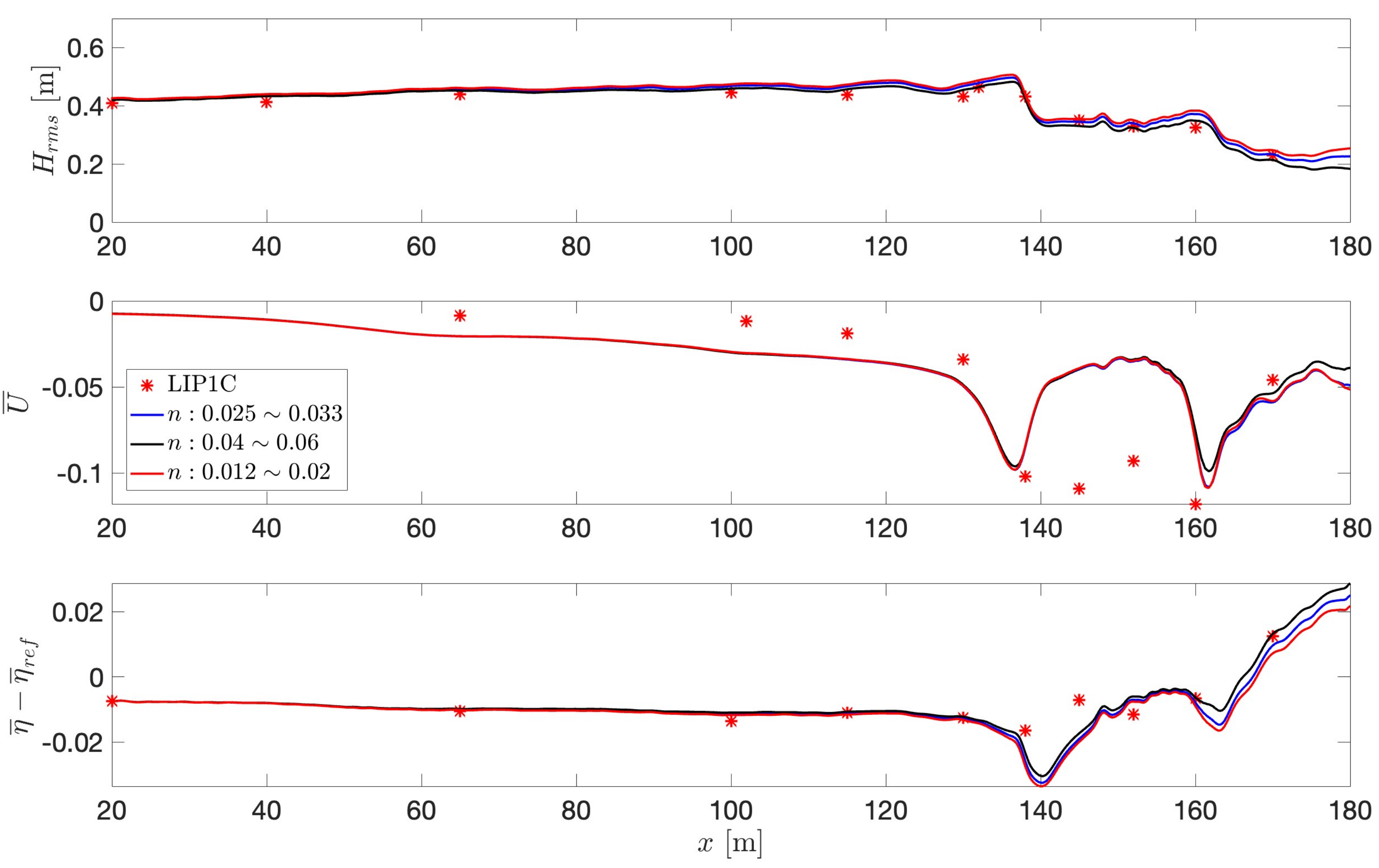
$\overline{U}$: Time-averaged horizontal velocity

$\overline{\eta}$: Time-averaged amplitude

More energetic



Less energetic



Negative $\overline{U}$: compensated by undertow effect

Open question & Perspective

Is there a better numerical scheme for bed evolution equation that can bypass the use of k ?

Perspectives

- Implementation in TOLOSA with new numerical scheme developed by Richard & Duran, **2024** for hydrodynamic model
- Bed evolution in the LIP experiment to validate the model's capability in capturing sandbar migration.
- Systematic derivation of the sediment transport equations

Merci de votre attention